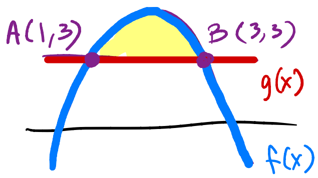
Last updated 3/3/22

1. Let $f(x) = 4x - x^2$ and $g(x) = 3$.
   (a) Find the coordinates of A and B, the points of intersection of f and g.
   (b) Calculate the area enclosed between the curve and the line.



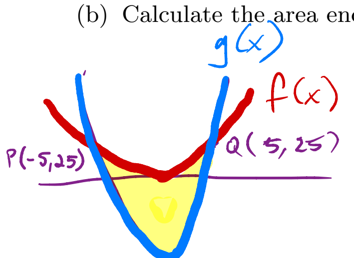
$4x - x^2 = 3$
$x^2 - 4x + 3 = 0$
$(x-1)(x-3)$
$x = 1, 3$

$$\int_1^3 f(x) - g(x)\,dx$$

$$2x^2 - \frac{x^3}{3} - 3x \Big|_1^3$$

$$(18 - 9 - 9) - (2 - \tfrac{1}{3} - 3) = \frac{4}{3}$$

2. Let $f(x) = x^2$ and $g(x) = 2x^2 - 25$.
   (a) Find the coordinates of P and Q, the points of intersection of f and g.
   (b) Calculate the area enclosed between the curves.



$2x^2 - 25 = x^2$
$x^2 - 25 = 0$
$(x+5)(x-5) = 0$

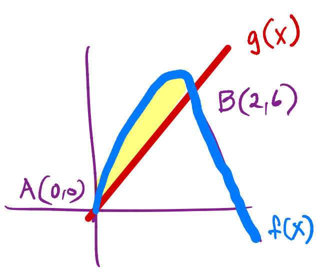
$$\int_{-5}^{5} x^2 - (2x^2 - 25)\,dx$$

$$\int_{-5}^{5} -x^2 + 25\,dx$$

$$-\frac{x^3}{3} + 25x \Big|_{-5}^{5} = \left(-\frac{125}{3} + 125\right) - \left(\frac{125}{3} - 125\right)$$

$$= \frac{500}{3}$$

3. Let $f(x) = 7x - 2x^2$ and $g(x) = 3x$.
   (a) Find the coordinates of A and B, the points of intersection of f and g.
   (b) Calculate the area enclosed between the curve and the line.



$7x - 2x^2 = 3x$
$2x^2 - 4x = 0$
$2x(x-2) = 0$

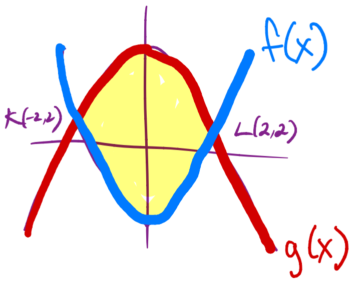
$$\int_0^2 7x - 2x^2 - 3x\,dx$$

$$\int_0^2 4x - 2x^2\,dx$$

$$2x^2 - \frac{2x^3}{3} \Big|_0^2$$

$$8 - \frac{16}{3} = \frac{8}{3}$$

4. Let $f(x) = 2x^2 - 6$ and $g(x) = 10 - 2x^2$.
   (a) Find the coordinates of K and L, the points of intersection of f and g.
   (b) Calculate the area enclosed between the curves.

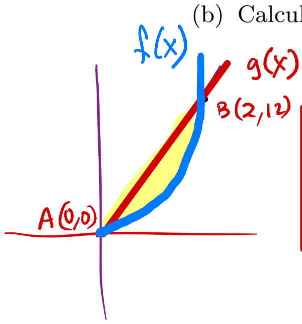


$$\boxed{\begin{aligned} 2x^2-6 &= 10-2x^2 \\ 4x^2-16 &= 0 \\ (2x-4)(2x+4) &= 0 \end{aligned}}$$

$$\int_{-2}^{2} (10-2x^2)-(2x^2-6)\,dx$$

$$\int_{-2}^{2} 16-4x^2\,dx = 16x-\frac{4x^3}{3}\Big|_{-2}^{2} = 32-\frac{32}{3})-(-32+\frac{32}{3}) = 64-\frac{64}{3} = \frac{128}{3}$$

5. Let $f(x) = x^3 + x^2$ and $g(x) = 2x^2 + 2x$.
   (a) Find the coordinates of A and B, the points of intersection of f and g in the first quadrant.
   (b) Calculate the area enclosed between the curves in the first quadrant.



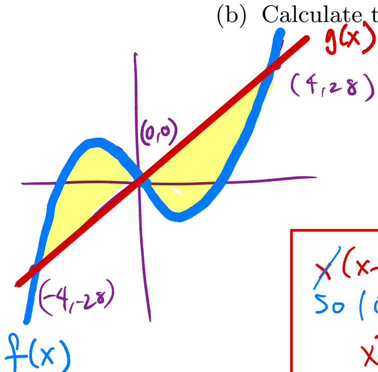
$$\boxed{\begin{aligned} x^3+x^2 &= 2x^2+2x \\ x^2(x+1) &= 2x(x+1) \\ (x^2-2x)(x+1) &= 0 \\ x(x-2)(x+1) &= 0 \end{aligned}}$$

$$\int_0^2 (2x^2+2x)-(x^3+x^2)\,dx$$

$$\int_0^2 -x^3+x^2+2x\,dx$$

$$-\frac{x^4}{4}+\frac{x^3}{3}+x^2\Big|_0^2 = (-\frac{16}{4}+\frac{8}{3}+4)-0 = \frac{8}{3}$$

6. Let $f(x) = x(x-3)(x+3)$ and $g(x) = 7x$.
   (a) Find the coordinates of A, B, and C, the points of intersection of f and g.
   (b) Calculate the area enclosed between the curve and the line.



$\cancel{x}(x-3)(x+3) = 7\cancel{x}$
So (0,0) is a solution
$x^2-9=7$
$x^2-16=0$
$(x+4)(x-4)=0$

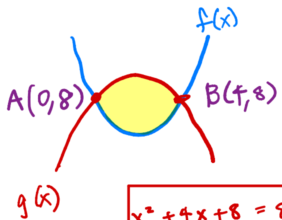
$$A = \int_{-4}^{0} f(x)-g(x)\,dx + \int_0^4 g(x)-f(x)\,dx$$

$$= \int_{-4}^{0} (x^3-9x)-7x\,dx + \int_0^4 7x-(x^3-9x)\,dx$$

$$= \int_{-4}^{0} x^3-16x\,dx + \int_0^4 16x-x^3\,dx$$

By symmetry $= 2\left[8x^2-\frac{x^4}{4}\right]_0^4 = 2\left[128-64\right] = 128$

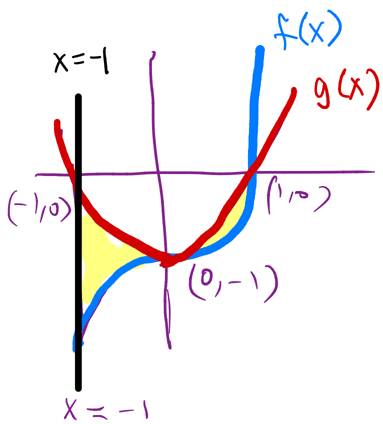
7. Let $f(x) = x^2 - 4x + 8$ and $g(x) = 8 + 4x - x^2$.
   (a) Find the coordinates of A and B, the points of intersection of f and g.
   (b) Calculate the area enclosed between the curves.



$$x^2 + 4x + 8 = 8 + 4x - x^2$$
$$2x^2 - 8x = 0$$
$$2x(x-4) = 0$$

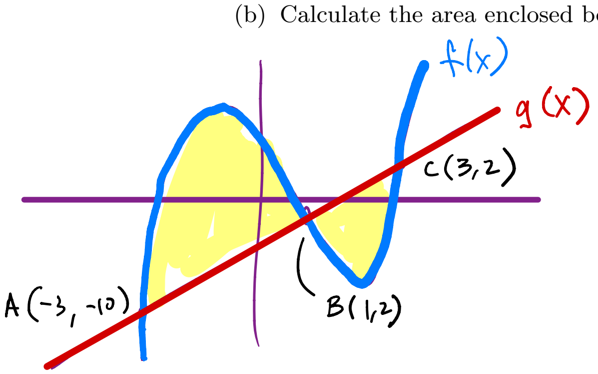
$$A = \int_0^4 g(x) - f(x)\,dx = \int_0^4 8x - 2x^2\,dx$$
$$= 4x^2 - \frac{2}{3}x^3 \Big|_0^4$$
$$= 64 - \frac{128}{3} = \frac{64}{3}$$

8. Let $f(x) = x^3 - 1$ and $g(x) = x^2 - 1$.
   (a) Calculate the area enclosed between the curves and the lines $x = -1$ and $x = 1$.



$$\int_{-1}^{1} (x^2-1) - (x^3-1)\,dx$$
$$\int_{-1}^{1} x^2 - x^3\,dx = \frac{x^3}{3} - \frac{x^4}{4}\Big|_{-1}^{1}$$
$$= \left(\frac{1}{3} - \frac{1}{4}\right) - \left(\frac{-1}{3} - \frac{1}{4}\right) = \frac{2}{3} + 0 = \frac{2}{3}$$

9. Let $f(x) = x^3 - x^2 - 7x + 5$ and $g(x) = 2x - 4$. 4l, 3
   (a) Find the coordinates of A, B, and C, the points of intersection of f and g.
   (b) Calculate the area enclosed between the curves.

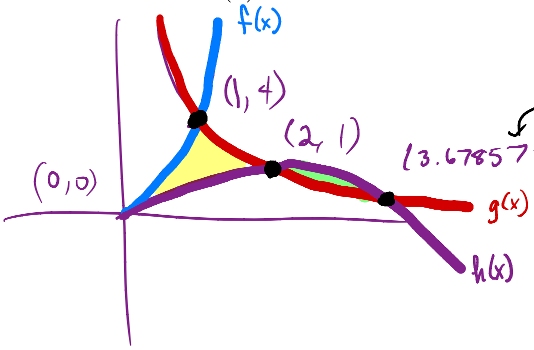


$$\int_{-3}^{1} f(x) - g(x)\,dx + \int_{1}^{3} g(x) - f(x)\,dx$$
$$\int_{-3}^{1} x^3 - x^2 - 9x + 9\,dx + \int_{1}^{3} -x^3 + x^2 + 9x - 9x\,dx$$
$$\frac{x^4}{4} - \frac{x^3}{3} - \frac{9}{2}x^2 + 9x\Big|_{-3}^{1} + \left(-\frac{x^4}{4} + \frac{x^3}{3} + \frac{9}{2}x^2 - 9x\Big|_{1}^{3}\right)$$
$$\left(\frac{1}{4} - \frac{1}{3} - \frac{9}{2} + 9\right) - \left(\frac{3^4}{4} + \frac{3^3}{3} - \frac{81}{2} - 27\right) + \left(-\frac{3^4}{4} + \frac{3^3}{3} + \frac{81}{2} - 27\right) - \left(-\frac{1}{4} + \frac{1}{3} + \frac{9}{2} - 9\right)$$
$$\left(\frac{53}{12}\right) - \left(\frac{-153}{4}\right) + \left(\frac{9}{4}\right) - \left(\frac{53}{12}\right)$$
$$\frac{128}{3} + \frac{20}{3} = \frac{148}{3}$$

10. Let $f(x) = x(x+3)$, $g(x) = \dfrac{4}{x^2}$ and $h(x) = x - \dfrac{x^2}{4}$.

(a) Find the coordinates of A, B, C, and D, the points of intersection of f, g and h in the first quadrant.

(b) Calculate the area enclosed between the curves.



sto> A

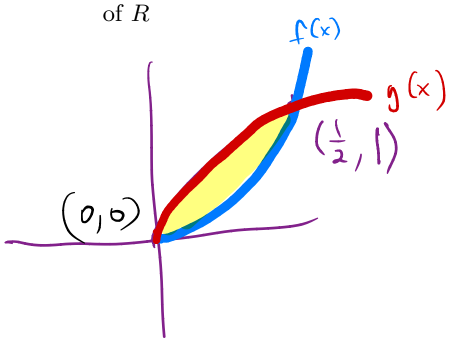
(3.67857351, 0.29559)

$$\int_0^1 f(x) - h(x)\,dx + \int_1^2 g(x) - h(x)\,dx + \int_2^A h(x) - g(x)\,dx$$

$$\frac{17}{12} + \frac{13}{12} + 0.3718212$$

$$2.8718$$

11. Let R be the region in the first quadrant enclosed by the graphs of $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$ Find the area of R



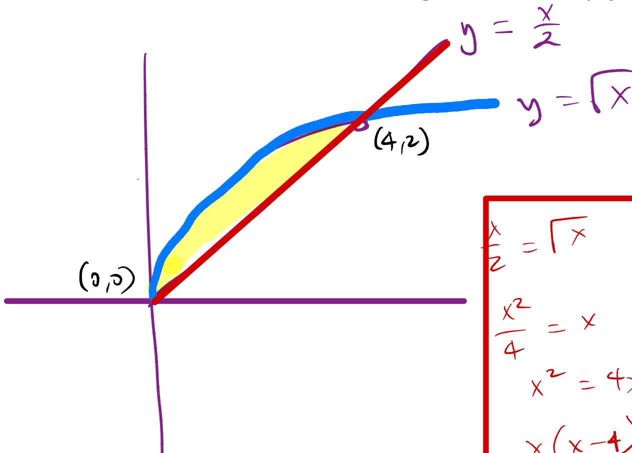
$$\int_0^{1/2} \sin \pi x - 8x^3\,dx$$

$$\left. \frac{-1}{\pi} \cos \pi x - 2x^4 \right|_0^{1/2}$$

$$\left(0 - 2\left(\frac{1}{2}\right)^4\right) - \left(\frac{-1}{\pi}\right)$$

$$-\frac{1}{8} + \frac{1}{\pi} \approx 0.1933698862$$

12. Find the area of the region bounded by $y = \sqrt{x}$ and $y = \dfrac{x}{2}$



$$\frac{x}{2} = \sqrt{x}$$

$$\frac{x^2}{4} = x$$

$$x^2 = 4x$$

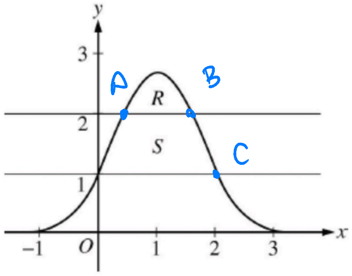
$$x(x-4) = 0$$

$$\int_0^4 \sqrt{x} - \frac{x}{2}\,dx$$

$$\left. \frac{2}{3}x^{3/2} - \frac{x^2}{4} \right|_0^4$$

$$\frac{16}{3} - \frac{4 \cdot 3}{3} = \frac{4}{3}$$

13. Let $f(x) = e^{2x-x^2}$. Let region R be the area bounded by f and above the horizontal line $y = 2$, and let S be region bounded by the graph of f and between the horizontal lines $y = 1$ and $y = 2$ Find the area of R and S.



$$R = \int_{A = 0.44605\,70251}^{B = 1.553942975} f(x) - 2 \, dx = 0.514142785 6$$

$$S = \int_0^{C=2} f(x) - 1 \, dx - R$$

$$S = 2.060156939 - R$$

$$S = 1.546014153$$

14. Let R be the region in the first quadrant under the graph of $y = \dfrac{1}{\sqrt{x}}$ for $4 \le x \le 9$.

(a) Find the area of R

$$\int_4^9 x^{-1/2} \, dx = 2x^{1/2} \Big|_4^9$$

$$= 2(3-2)$$

$$= 2$$

(b) If the line $x = k$ divided the region R into two regions of equal area, what it the value of k?

$$\int_4^k x^{-1/2} \, dx = 2x^{1/2} \Big|_4^k$$

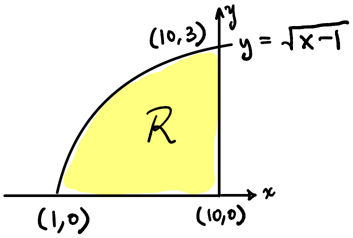
$$2\sqrt{k} - 4 = 1 \quad (\text{half of } 2)$$

$$\sqrt{k} = \frac{5}{2}$$

$$k = \frac{25}{4}$$

15. Let R be the region enclosed by the graph of $y = \sqrt{x-1}$, the vertical line $x = 10$ and the x-axis.

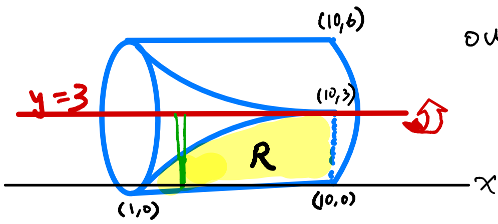
(a) Find the area of R



$$\int_1^{10} (x-1)^{1/2}\,dx = \frac{2}{3}(x-1)^{3/2}\Big|_1^{10}$$

$$= \frac{2}{3}\left(9^{3/2} - 0\right) = 2\cdot\frac{3^3}{3^1} = 18$$

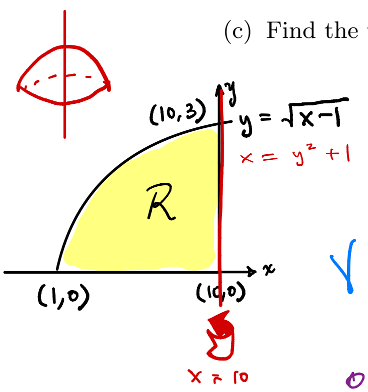
(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 3$.



outer: $R(x) = 3$ inner: $r(x) = 3 - \sqrt{x-1}$

$$\pi\int_1^{10} [R(x)]^2 - [r(x)]^2\,dx = 212.0575058$$

(c) Find the volume of the solid generated when R is revolved about the vertical line $x = 10$.



$$R(y) = 10 - (y^2+1) = 9 - y^2$$

$$R^2 = 81 - 18y^2 + y^4$$

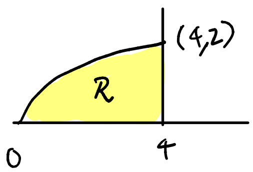
$$V = \pi\int_0^3 [R(y)]^2\,dy = 407.1504079$$

or

$$V = \pi\int_0^3 81 - 18y^2 + y^4\,dy = \pi\left[81y - 6y^3 + \frac{y^5}{5}\right]_0^3 = \frac{648}{5}\pi$$

16. (No Calc) Let R be the region bounded by the x-axis, the graph of $y = \sqrt{x}$ and the vertical line $x = 4$

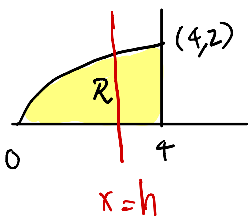
(a) Find the area of R.



$$\int_0^4 \sqrt{x}\,dx = \frac{2}{3} x^{3/2} \Big|_0^4$$

$$= \frac{2}{3}\left[4^{3/2} - 0\right] = \frac{2}{3} 2^3 = \frac{16}{3}$$

(b) Find the value of h such that the vertical line $x = h$ divided the region R into two regions of equal area.



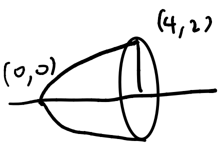
$$\int_0^h \sqrt{x}\,dx = \frac{2}{3} x^{3/2}\Big|_0^h = \frac{2}{3}\left[h^{3/2} - 0\right]$$

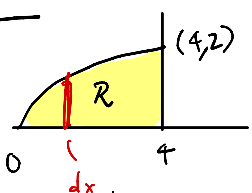
$$\frac{2h^{3/2}}{3} = \frac{8}{3} \qquad \left(\frac{1}{2} \text{ of } \frac{16}{3}\right)$$

$$h^{3/2} = 4$$

$$h = 4^{2/3} \text{ or } \sqrt[3]{16}$$

(c) Find the volume of the solid generated when R is revolved around the x-axis.





$$V = \pi \int_a^b R(x)^2\,dx$$

$$\pi \int_0^4 \left(\sqrt{x}\right)^2 dx = \pi \int_0^4 x\,dx$$

$$= \pi \frac{x^2}{2}\Big]_0^4 = 8\pi$$

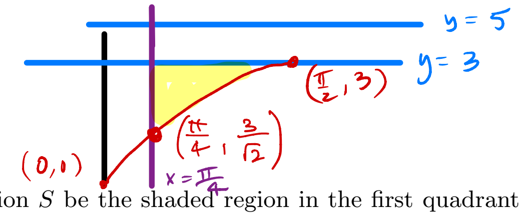
(d) The vertical line $x = k$ divides the region R into two regions such that when these two regions are revolved about the x-axis, they generated solids with equal volumes. Find the value k.

$$\pi \int_0^k \left(\sqrt{x}\right)^2 dx = \frac{\pi}{2} x^2 \Big|_0^k$$

$$\frac{\cancel{\pi}}{2} k^2 = 4\cancel{\pi}$$

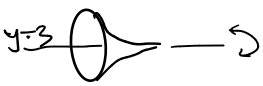
$$k^2 = 4 \cdot 2$$

$$k = \sqrt{8} \text{ or } 2\sqrt{2}$$



17. Let the region S be the shaded region in the first quadrant bounded above by the horizontal line $y = 3$, below by the graph of $y = 3\sin x$, and on the left by the vertical line $x = \frac{\pi}{4}$.

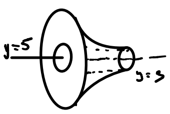
(a) What is the volume of the solid generated if S is revolved about the horizontal line $y = 3$?



$R(x) = 3 - 3\sin x$

$\pi \int_{\pi/4}^{\pi/2} [R(x)]^2 \, dx = 0.3925518808$

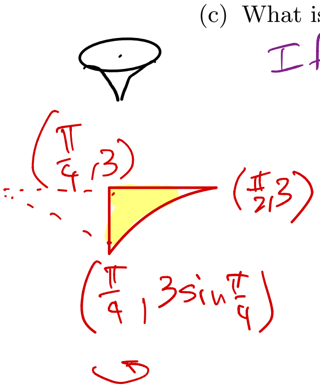
(b) What is the volume of the solid generated if S is revolved about the horizontal line $y = 5$?



outer $R(x) = 5 - 3\sin x$ inner $r(x) = 5 - 3 = 2$

$\pi \int_{\pi/4}^{\pi/2} [R(x)]^2 - [r(x)]^2 \, dx = 8.278869656$

(c) What is the volume of the solid generated if S is revolved about the vertical line $x = \frac{\pi}{4}$?



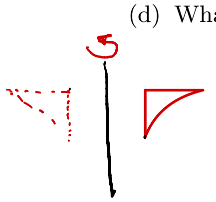
If $y = 3\sin x$, then $x = \arcsin\left(\frac{y}{3}\right)$

$R(y) = \arcsin\left(\frac{y}{3}\right) - \frac{\pi}{4}$

$\pi \int_{3/\sqrt{2}}^{3} (R(y))^2 \, dy = 0.292769773$

$\left(3\sin\frac{\pi}{4} = \frac{3\sqrt{2}}{2} = \frac{3}{\sqrt{2}} \approx 2.121320344\right)$

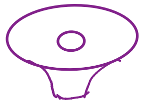
(d) What is the volume of the solid generated if S is revolved about the y-axis?



outer $R(y) = \arcsin\left(\frac{y}{3}\right)$

inner $r(y) = \frac{\pi}{4}$

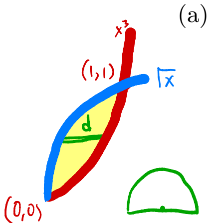
$\pi \int_{3/\sqrt{2}}^{3} [R(y)]^2 - [r(y)]^2 \, dy = 1.451827223$



so $x = y^2$ so $x = y^{1/3}$

18. The base of a solid is bounded by $\boxed{y = \sqrt{x}}$ and $\boxed{y = x^3}$. Find the volume of the solid with each of the following cross sections:

(a) Semi-circles perpendicular to the y-axis.



$A = \frac{\pi}{2}\left(\frac{d}{2}\right)^2 = \frac{\pi}{8}d^2$

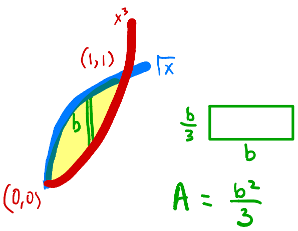
$d = y^{1/3} - y^2$ (Right is larger than left)

$$V = \frac{\pi}{8}\int_0^1 \left(y^{1/3} - y^2\right)^2 dy = \frac{\pi}{8}\int_0^1 y^{2/3} - 2y^{7/3} + y^4 \, dy$$

$$V = \frac{\pi}{8}\left(\frac{3}{5}y^{5/3} - \frac{6}{10}y^{10/3} + \frac{y^5}{5}\right)\Big|_0^1 = \frac{\pi}{8}\left(\frac{3}{5} - \frac{6}{10} + \frac{1}{5}\right)$$

$$= \frac{\pi}{40}$$

(b) Rectangles perpendicular the the x-axis whose height is $\frac{1}{3}$ their base.



$A = \frac{b^2}{3}$

$$\int_0^1 \frac{b^2}{3}\,dx = \frac{1}{3}\int_0^1 \left(\sqrt{x} - x^3\right)^2 dx = \frac{1}{3}\int_0^1 x - 2x^{7/2} + x^6 \, dx$$

$$= \frac{1}{3}\left(\frac{x^2}{2} - \frac{4}{9}x^{9/2} + \frac{x^7}{7}\right)\Big|_0^1$$

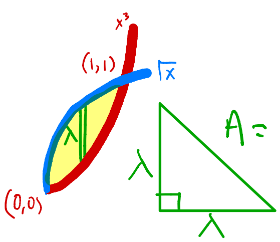
$$= \frac{1}{3}\left(\frac{25}{126}\right) = \frac{25}{378} \quad (\approx .0661)$$

(c) Isosceles Right triangle perpendicular to the x-axis with a leg in the base.

$$V = \int_0^1 A(x)\,dx = \frac{1}{2}\int_0^1 \left(\sqrt{x} - x^3\right)^2 dx = \frac{1}{2}\int_0^1 x - \frac{4}{9}x^{9/2} + \frac{x^7}{7}\,dx$$

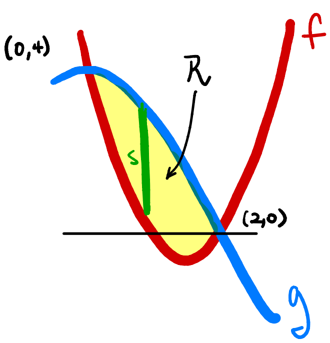
$$= \frac{1}{2}\left(\frac{x^2}{2} - \frac{4}{9}x^{9/2} + \frac{x^7}{7}\right)\Big|_0^1 = \frac{1}{2}\left(\frac{25}{126}\right) = \frac{25}{252}$$

$(\approx .0992)$



$A = \frac{\lambda^2}{2}$

19. Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{\pi x}{4}\right)$. Let R be the region bounded by the graphs of f and g. The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. What is the volume of this solid.



$A = s^2$

$A = \left(g(x) - f(x)\right)^2$

$$V = \int_0^2 \left[g(x) - f(x)\right]^2 dx$$

$$= 8.503140149$$